Confinement of monopole using flux string[1]

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Abstract

We study the confinement of fermionic magnetic monopoles by a thin flux tube of the Abelian Higgs model. Parity demands that the monopole currents be axial. This implies that the model is consistent only if there are at least two species of fermions being confined.

In Nambu's model of confinement [1], quarks are the endpoints of an open Dirac string [2]. For massive gauge fields these strings become real [3], unlike the usual Dirac string. We take fermionic monopoles and attach them to the ends of a flux string such that no flux can escape. To do this, we first dualize the theory to write it in terms of a 'magnetic' gauge field and an antisymmetric tensor. Then we minimally couple the monopole current to the 'magnetic' photon and redualize the theory. The result is a theory of magnetic flux tubes interacting with a massive Abelian vector gauge field. The tubes are sealed at the ends by fermions, thus providing a toy model for quark confinement.

We start with the generating functional for the Abelian Higgs model in $3+1$ dimensions, coupled to an Abelian gauge field A^e_μ. The partition function is given by $Z = \int \mathscr{D}A^e_\mu \mathscr{D}\Phi \mathscr{D}\Phi^* \exp iS$, where

$$S = \int d^4x \left(-\frac{1}{4} F^e_{\mu\nu} F^{e\mu\nu} + |D_\mu \Phi|^2 + \lambda(|\Phi|^2 - v^2)^2 \right), \tag{1}$$

where $D_\mu = \partial_\mu + ieA^e_\mu$, and $F^e_{\mu\nu} = \partial_\mu A^e_\nu - \partial_\nu A^e_\mu$ is the Maxwell field strength.

We will consider the theory in the London limit $\lambda \to \infty, |\Phi| = v$. We change variables from Φ, Φ^* to the radial Higgs field ρ and the angular field θ. Then the action becomes $\int d^4x \left(-\frac{1}{4} F^e_{\mu\nu} F^{e\mu\nu} + \frac{v^2}{2}(\partial_\mu \theta + eA^e_\mu)^2 \right)$. In the presence of flux tubes we can decompose θ into a regular and a singular part. The singular part θ_s is related to the world sheet Σ of the flux tube as $\epsilon^{\mu\nu\rho\lambda}\partial_\rho\partial_\lambda\theta^s = \Sigma^{\mu\nu}$, where

$$\Sigma^{\mu\nu} = 2\pi n \int_\Sigma d\sigma^{\mu\nu}(x(\xi))\, \delta^4(x - x(\xi)), \tag{2}$$

with $\xi = (\xi^1, \xi^2)$ the coordinates on the world-sheet, $d\sigma^{\mu\nu}(x(\xi)) = \epsilon^{ab}\partial_a x^\mu \partial_b x^\nu$, and $2\pi n$ is the vorticity [4].

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We can rewrite the action using standard techniques of linearization [5, 6, 7]. Then integrating over θ_r and A^e_μ we get the partition function as

$$\int \mathscr{D}A^m_\mu \mathscr{D}x_\mu(\xi)\mathscr{D}B_{\mu\nu} \exp i \int \left[-\frac{1}{4}(eB_{\mu\nu} + \partial_{[\mu}A^m_{\nu]})^2 + \frac{1}{12v^2}H^2_{\mu\nu\rho} - \frac{1}{2}\Sigma_{\mu\nu}B^{\mu\nu} \right] , \tag{3}$$

where $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$. What we have achieved here is the dualization of the photon A^e_μ to the 'magnetic photon' A^m_μ. We have also replaced the integration measure $\mathscr{D}\theta^s$ by $\mathscr{D}x_\mu(\xi)$. Here $x_\mu(\xi)$ parametrizes the surface on which the field θ is singular. The Jacobian for this change of variables gives the action for the string on the background spacetime [8, 9].

The equation of motion for the field $B_{\mu\nu}$ is $\partial_\lambda H^{\lambda\mu\nu} = -(m^2/e)\, G^{\mu\nu} - m^2\, \Sigma^{\mu\nu}$, where $G_{\mu\nu} = eB_{\mu\nu} + \partial_\mu A^m_\nu - \partial_\nu A^m_\mu$, and $m = ev$. Absence of a magnetic current gives $\partial_\mu G^{\mu\nu} = 0$, so from the above equation of motion we get $\partial_\nu \Sigma^{\mu\nu} = 0$. This equation means that in the absence of magnetic monopoles, the vorticity current tensor $\Sigma_{\mu\nu}$ is conserved. Therefore in the absence of monopoles, flux tubes are closed or infinite.

Now consider a massless fermionic monopole current minimally coupled to the magnetic or dual photon. This current is axial in order to conserve the parity of Maxwell's equations. However, a theory containing axial currents is anomalous. We can cancel the anomaly by introducing another species of fermionic monopoles with opposite charge. Writing the two species as q and q', with magnetic charges $+g$ and $-g$, we can write the current as $j^\mu_m = g\bar{q}\gamma_5\gamma^\mu q - g\bar{q}'\gamma_5\gamma^\mu q'$.

The partition function of Eq. (3) is modified to include the fermionic monopoles, minimally coupled to the 'magnetic photon' A^m_μ, so the Lagrangian now reads $\mathscr{L} = -\frac{1}{4}(eB_{\mu\nu} + \partial_{[\mu}A^m_{\nu]})^2 + (1/12v^2)H^2_{\mu\nu\rho} - \frac{1}{2}\Sigma_{\mu\nu}B^{\mu\nu} + i\bar{q}\partial\!\!\!/ q + i\bar{q}'\partial\!\!\!/ q' - A^m_\mu j^\mu_m$. The conservation condition is modified to $(1/e)\partial_\nu\Sigma^{\mu\nu} = j^\mu_m$.

We can see that this equation is a consequence of gauge invariance. We take a transformation $B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu$, $A^m_\mu \to A^m_\mu - (k/g)\Lambda_\mu$. The Lagrangian is invariant if we set $eg = k$. The flux due to the monopole is $4\pi g$. Since this flux is fully confined inside the tube, we see by using the conservation condition and $eg = k$ that, $2n\pi =$ vorticity flux $= (k/g)$ (monopole flux) $= 4\pi k$. So $k = (n/2)$, and we must have the Dirac quantization condition $eg = (n/2)$. Since the transformation given above is only a change of variables, Z cannot depend on Λ_μ. Thus Λ_μ can be integrated out with no effect other than the introduction of an irrelevant constant factor in Z, which we ignore. After integrating over Λ_μ, we get [10]

$$Z = \int \mathscr{D}A^m_\mu \cdots \delta\Big[\frac{1}{e}\partial_\mu\Sigma^{\mu\nu} + j^\nu_m\Big] \exp i \int d^4x \left[-\frac{1}{4}(eB_{\mu\nu} + \partial_{[\mu}A^m_{\nu]})^2 + \frac{1}{12v^2}H^2_{\mu\nu\rho} \right.$$
$$\left. -\frac{1}{2}\Sigma_{\mu\nu}B^{\mu\nu} + i\bar{q}\partial\!\!\!/ q + i\bar{q}'\partial\!\!\!/ q' - A^m_\mu j^\mu_m \right] . \tag{4}$$

One can see from the δ-functional that the vorticity current tensor is not conserved, but is cancelled by the fermionic current. So the strings are open strings with

fermions stuck at the ends. Now we dualize the theory a second time and get back to a vector gauge field. Then the partition function becomes

$$\begin{aligned} Z &= \int \mathscr{D}x_\mu(\xi)\mathscr{D}B_{\mu\nu}\mathscr{D}A_\mu \cdots \delta\Big[\frac{1}{e}\partial_\mu\Sigma^{\mu\nu} + j_m^\nu\Big] \exp\, i\int d^4x\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{1}{12v^2}H_{\mu\nu\rho}H^{\mu\nu\rho} + \frac{1}{2g}\epsilon^{\mu\nu\rho\lambda}B_{\mu\nu}\partial_\rho A_\lambda + i\bar{q}\partial\!\!\!/ q + i\bar{q}'\partial\!\!\!/ q'\Big]. \end{aligned} \tag{5}$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - (1/2e)\epsilon_{\mu\nu\sigma\lambda}\Sigma^{\sigma\lambda}$. The theory is now in the form we originally intended, and contains thin tubes of flux. The new feature is that the ends of the flux tube are *sealed* by fermions, so that no flux escapes.

The mass per unit length of these strings is easily found to be $\mu \sim g^2v^2\lambda$. Such a string of finite length would collapse in order to minimize the energy unless it was stabilized by its angular momentum. For a rotating string of length l, energy per unit length μ, angular momentum J, the energy function is $E = \mu l + J^2/2\mu l^3$. We can see that for the stable flux tube with magnetic monopoles at the ends, $J/E^2 = constant$, similar to the well-known Regge trajectory for mesons. The gauge field A_μ is massive, with mass $m = v/g$ [11, 12]. It does not couple directly to the fermionic monopoles at the ends. The δ-functional guarantees that the monopoles must seal the ends of the string. If we suggestively rename q and q' to u and $\bar{d}$, the allowed configurations are $u\bar{d}$, $\bar{u}d$, and $u\bar{u} \pm d\bar{d}$, which can couple to electroweak gauge fields also.

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